Modeling social behaviour in an uncertain environment, application in epidemiology

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2 Modelization of the problem

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Vaccine scares:

Influenza A (H1N1) (flu) (2009-10)

• At 15/06/2010 flu (H1N1): 18.156 deads in 213 countries (WHO)
• France: 1334 severe forms (out of 7.7M-14.7M people infected)

Vaccination in France

• Adjuvant suspected of some neurological undesired effects; mass vaccination uncertainty (few previous studies for this size)
• Very costly campaign (500M EUR),
• Low efficiency (8% to 10% in France with respect to e.g., 24% US or 74% Canada).
Previous vaccine scares (some have been disproved):

- France: hepatitis B vaccines cause multiple sclerosis
- US: mercury additives are responsible for the rise in autism
- UK: the whooping cough (1970s), the measles-mumps-rubella (MMR) (1990s).

**Vaccine Scares**: “as cases of a disease decrease, people become complacent about their risk, and the threat of vaccines (imagined or real) seems greater than the threat of disease” (C. Bauch)

**Question**: individual decisions sum up to give a global response. How to model this?
Outline

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General SIR model

\begin{align*}
    dS &= (\mu(1 - S) - \beta SI)\, dt - dV(t) \\
    dl &= (-\mu l + \beta SI - \gamma l)\, dt \\
    dR &= (-\mu R + \gamma l)\, dt + dV(t)
\end{align*}

With:

- $\mu$: rates of birth / death,
- $\beta$: probability of contamination,
- $\gamma$: rates of healing,
- $dV(t)$: measure of vaccination, several possibilities
  \begin{align*}
    dV(t) &= \lambda(t)S(t)\, dt : \text{probability of individual vaccination} \\
    \lambda(t) &\in [0, \lambda_{\text{max}}] \\
    dV(t) &= u(t)\, dt : \text{speed of vaccination } u(t) \in [0, u_{\text{max}}] \\
    \text{General case: } dV(t) &\text{ is a (positive) measure on } [0, \infty]
  \end{align*}

Number / proportion of individuals vaccinated up to time "t" is $\int_{0}^{t} dV(s)$ increasing.
Subsequently, we denote $X = (S, I)^T$ because $S_0 + I_0 + R_0 = 1$. 
Modelization of the cost

Cost for an infected person: \( r_I \)
Cost for a vaccinated person: \( r_V \)

Global cost for the society:

\[
J(X_0, V) = \int_0^\infty \beta S I r_I dt + \int_0^\infty r_V dV(t) \tag{2}
\]

With \( X_0 = (S(0), I(0))^T \)

It is an optimal control problem.
The value function of this problem is:

\[
\mathcal{V}(X) = \min_{w \in \Omega} J(X, w)
\]

And \( \Omega \) is the set of admissible functions.
Problem’s particularities

The value function $\mathcal{V}$ must satisfy the HJB equation:

$$-\mathcal{H}(X, \nabla \mathcal{V}) = 0$$

Let $X = (x_1; x_2)^T$ and

$$f(X, w) = (\mu(1 - x_1) - \beta x_1 x_2 - w; -\mu x_2 + \beta x_1 x_2 - \gamma x_2)$$

Then

$$\mathcal{H}(X, p) = \min_{w \in [0, u_{max}]} [f(X, w) \cdot p]$$

$$= -u_{max}(p_1 - r_v)_+ + \beta x_1 x_2 (r_l + p_2 - p_1) - \gamma x_2 p_2 \text{ for } \mu = 0$$

But there is no a priori certainty that the solutions are $C^1$ (possible discontinuity introduced by $V$)

We use the concept of viscosities solutions introduced by Pierre-Louis Lions. Widely used for the optimal control problem.
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For $\mu = 0$, already encounter technical problems.

At the boundary $I = 0$ there is no natural boundary condition to use. If the system starts with $I = 0$ it will remain with $I = 0$ at all times but this behavior is unstable! As soon as $I(0) > 0$ (even very very small) and $S(0) > \gamma/\beta$ the value functions takes very large values (larger than $S(0) - \max(r_I, r_V)\gamma/\beta$) and do not converge to zero when $I(0)$ tends to 0.
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Other problems

The cost function has no damping term.
Work in infinite horizon.
In general, a convenient hypothesis (cf. also Crandall, Ishii, Lions [1992]) is:

\[ \mathcal{H}(u, r) \leq \mathcal{H}(u, s) \quad \forall r \leq s \]

This is not our case.

Furthermore, the value function is independent of the time, it is a problem for uniqueness.
• Horst Behncke: "Optimal control of deterministic epidemics" use an optimal policy "all or nothing" (for a certain period in order to stop the epidemic). Do not use HJB. Passage to the limit inconclusive ($T \to \infty$).

• Alexei B. Piunoskiy et Damian Clancy: "An explicit optimal intervention policy for a deterministic epidemic model" supposes that the solution is $C^1$ so that they are perhaps suboptimal. Problem also when $\lambda_{max} \to \infty$. 
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Our contribution

Using viscosity solutions allows to:

- Prove existence and uniqueness of the solution
- Show that the solution is $C^1$
- Characterize the solution
- Is compatible with the limit $u_{\text{max}} \to \infty$ (and also $\lambda_{\text{max}} \to \infty$)

Existence of the following level value of $r_V$. If $r_I = 1$,

- $r_V < 1$: several types of solution
- $1 \leq r_V \leq 2$: optimal to vaccinate a few people, but sub-optimal for an individual
- $r_V > 2$: no vaccination
Current work and perspectives

Let $\mu \neq 0$

Try with other contact form (such as $\frac{\beta SI}{S+I}$).

Comparison of effects of optimal policies (global or individual).

When individual policy, the disease never finished.