Existence and Stability of Viscous Shock Profiles for Isentropic MHD with Infinite Electrical Resistivity

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Overview

- Isentropic gas
- Parallel MHD
- MHD with infinite electrical resistivity
- Known instabilities in multi-D problem
- Phase space
- Stability setup
- Numerical approximation

Equations

Magnetohydrodynamics (MHD)

$$\begin{cases} v_t - u_{1x} = 0, \\ u_{1t} + (p + (1/2\mu_0)(B_2^2 + B_3^2))_x = (((2\mu + \eta)/v)u_{1x})_x, \\ u_{2t} - ((1/\mu_0)IB_2)_x = ((\mu/v)u_{2x})_x, \\ u_{3t} - ((1/\mu_0)IB_3)_x = ((\mu/v)u_{3x})_x, \\ (vB_2)_t - (Iu_2)_x = ((1/\sigma\mu_0 v)B_{2x})_x, \\ (vB_3)_t - (Iu_3)_x = ((1/\sigma\mu_0 v)B_{3x})_x, \end{cases}$$

Hyperbolic wherever p is monotone.

Rescaled Equations

$$(v, u, \mu_0, x, t, B, a) \to \left(\frac{v}{\epsilon}, -\frac{u}{\epsilon s}, \epsilon\mu_0, -\epsilon s(x - st), \epsilon s^2 t, -\frac{B}{s}, \frac{a\epsilon^{-\gamma-1}}{s^2}\right)$$

$$\begin{cases} v_t + v_x - u_{1x} = 0 \\ u_{1t} + u_{1x} + \left(av^{-\gamma} + \left(\frac{1}{2\mu_0}\right)\left(B_2^2 + B_3^2\right)\right)_x = (2\mu + \eta)\left(\frac{u_{1x}}{v}\right)_x \\ u_{2t} + u_{2x} - \left(\frac{1}{\mu_0} IB_2\right)_x = \mu\left(\frac{u_{2x}}{v}\right)_x \\ u_{3t} + u_{3x} - \left(\frac{1}{\mu_0} IB_3\right)_x = \mu\left(\frac{u_{3x}}{v}\right)_x \\ (vB_2)_t + (vB_2)_x - (Iu_2)_x = \left(\left(\frac{1}{\sigma\mu_0 v}\right) B_{2x}\right)_x \\ (vB_3)_t + (vB_3)_x - (Iu_3)_x = \left(\left(\frac{1}{\sigma\mu_0 v}\right) B_{3x}\right)_x \end{cases}$$

Profile Equations

Traveling wave solution: $(v, u_1, u, B)(x,t) = (v, u_1, \tilde{u}, \tilde{B})(x - st)$

$$(v, u_1, \tilde{u}, \tilde{B})(\pm\infty) = (v, u_1, \tilde{u}_2, \tilde{B})_\pm.$$

$$\begin{cases} v' - u_1' = 0, \\ u_1' + (p + (1/2\mu_0)|\tilde{B}|^2)' = (((2\mu+\eta)/v)u_1')' \\ \tilde{u}' - ((1/\mu_0)I\tilde{B})' = ((\mu/v)\tilde{u}')' \\ (v\tilde{B})' - (I\tilde{u})' = ((1/\sigma\mu_0 v)\tilde{B}')' \end{cases}$$

Profile Equations

$u = u_1, \; w := \tilde{u}, \; B = \tilde{B}$

$$(2\mu + \eta)v' = v(v-1) + v(p - p_-) + \frac{v}{2\mu_0}(B^2 - B_-^2),$$
$$\mu w' = vw - \frac{vI}{\mu_0}(B - B_-),$$
$$\frac{1}{\sigma\mu_0}B' = v^2 B - vB_- - Ivw,$$

with $u \equiv v - 1$.

Profile Equations, $\sigma = \infty$

$$\sigma = \infty$$

$$(v\tilde{B})' - (I\tilde{u})' = 0,$$

$$(vB)' - (Iw)' = 0$$

$$B = \frac{B_- + Iw}{v}.$$

$$(2\mu + \eta)v' = v(v-1) + v(p - p_-) + \frac{1}{2\mu_0 v}((B_- + Iw)^2 - v^2 B_-^2),$$

$$\mu w' = vw - \frac{I}{\mu_0}\big(B_-(1-v) + Iw\big).$$

RH conditions

$$u = u_1, \; B = (B_2, B_3), \; w = (u_2, u_3),$$

$$\begin{aligned} -s[v] &= [u], \\ -s[u] &= -\left[p + \frac{B^2}{2\mu_0}\right], \\ -s[w] &= I\left[\frac{B}{\mu_0}\right], \\ -s[vB] &= I[w]. \end{aligned}$$

Important parameters

$$J := \frac{(B_{2-})^2}{2\mu_0} \text{ and } K := \frac{I^2}{\mu_0}.$$

(Note that, under the rescaling that we used, $I = -\frac{I}{s}$, $J = \frac{B_{2-}^2}{2\epsilon s^2 \mu_0} = \frac{B_{2-}^2}{2v_- s^2 \mu_0}$, $K = \frac{(I)^2}{\epsilon s^2 \mu_0} = \frac{(I)^2}{v_- s^2 \mu_0}$ in the original coordinates.)

Unique RH solution

$$u_+ = v_+ - 1, \quad B_{2_-} = \left(\frac{v_+ - K}{1 - K}\right) B_{2_+}, \quad w_+ = \frac{K}{I}\left(\frac{1 - v_+}{1 - K}\right) B_{2_+},$$

$$a = \left(\frac{1 - v_+}{v_+^{-\gamma} - 1}\right)\left(1 - \frac{B_{2_+}^2}{2\mu_0}\frac{(1 + v_+ - 2K)}{(1 - K)^2}\right) = \left(\frac{1 - v_+}{v_+^{-\gamma} - 1}\right)\left(1 - J\frac{(1 + v_+ - 2K)}{(v_+ - K)^2}\right).$$

This is physically meaningful if and only if $a > 0$, or

$$-1 < v_+ - 1 < 2(K - 1) + (1 - K)^2\left(\frac{2\mu_0}{B_{2_+}^2}\right).$$

(For $K \geq 1/2$, this gives no restriction. For $K < 1/2$, $B_{2_+}^2 < \frac{2\mu_0(1-K)^2}{1-2K}$ or $J < \frac{(v_+-K)^2}{1-2K}$.)

Shock type

Consider a general system of conservation laws

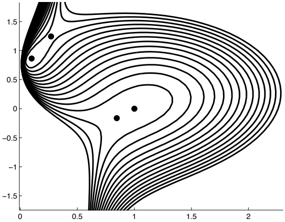
$$U_t + F(U)_x = (\mathcal{B}(U)U_x)_x, \quad U \in R^n$$

Inviscid shock waves correspond to triples (U_-, U_+, s) satisfying the Rankine–Hugoniot conditions

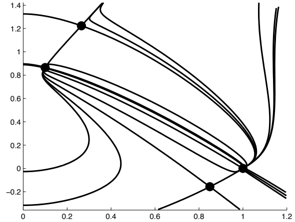
$$[F(U)] - s[U] = 0,$$

where $[h] := h(U_+) - h(U_-)$ denotes the jump in quantity h across the shock. The *type* of the shock wave is defined by the degree of compressivity

$$\ell := \dim \mathcal{U}(dF(U_-) - sI) + \dim \mathcal{S}(dF(U_+) - sI) - n,$$



(*a*)



(*b*)

Figure: Typical phase portrait for MHD with two variables and infinite electric resistivity ($\sigma = \infty$). Parameter values are $\gamma = 2$, $\tau_+ = 0.1$, $I = 0.7$, $B_{2+} = 0.7$, and $\mu_0 = 1$. In Figure (a) we plot level sets of ϕ and in Figure (b) we draw the phase portrait.

Phase portraits: MHD

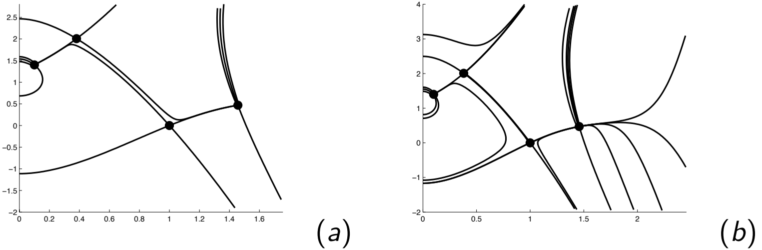


Figure: Transition to undercompressive profile. keeping $\tau = 2\mu + \eta = 1$ and letting $\mu \to 0$, we find that the overcompressive family is squeezed to an undercompressive connection somewhere between $\mu = 0.185$ (Figure (a)) and $\mu = 0.17$ (Figure (b)).

Spectral Stability

Linearization about the profile

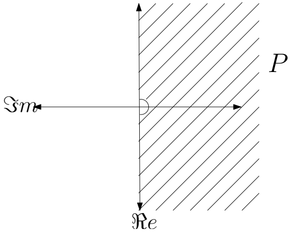
$$v_t = \underbrace{(d\mathcal{F}(\hat{u}) - s\partial_x)v}_{Lv} + \underbrace{Q(v, v_x, v_{xx}, \ldots)}_{\text{Higher order}}.$$

The eigenvalue problem is a BVP.

$$Lv = \lambda v, \quad v^{(k)}(\pm\infty) = 0.$$

Spectral Stability when $\sigma(L) \cap P = \emptyset$,

where $P = \{\lambda \mid \Re e(\lambda) \geq 0\} \backslash \{0\}$.



Nonlinear stability

Artificial Viscosity

Existence and Asymptotic Behavior
Madja and Pego (83)

Small-Amplitude Spectral Stability
Goodman (86)

Nonlinear Stability
T.P. Liu (86), partial result
Szepessy and Xin (92)

Pointwise Greens Function Bounds
T.P. Liu (97)

Spectral Stability $\Rightarrow$ Nonlinear Stability
Zumbrun and Howard (98,00)

Real Viscosity

Existence and Asymptotic Behavior
Pego (82)

Dissipativity Condition
Kawashima (83)

Small-Amplitude Spectral Stability (Navier Stokes)
Matsumura and Nishihara (Isentropic)(84)
Kawashima, Matsumura and Nishihara (86)

Small-Amplitude Spectral Stability
Humpherys and Zumbrun (02)

Spectral Stability $\Rightarrow$ Nonlinear Stability
Mascia and Zumbrun (01,04,06)

Eigenvalue Problem

Write the eigenvalue problem

$$\lambda v = Lv, \qquad -\infty < x < \infty,$$

as a first-order system

$$\begin{cases} W' = A(x,\lambda)W, & W \in \mathbb{C}^n \\ W(\pm\infty) = 0. \end{cases}$$

- Assume $A(x,\lambda)$ is consistently split in x.
- Assume $A(x,\lambda)$ is asymptotically constant in x, that is, $\lim_{x\to\pm\infty} A(x,\lambda) = A_{\pm}(\lambda)$.

Evans Function

Define at $x = 0$

$$D(\lambda) = \underbrace{W_1^+ \wedge \ldots \wedge W_k^+}_{S^+(\lambda)} \wedge \underbrace{W_{k+1}^- \wedge \ldots \wedge W_n^-}_{U^-(\lambda)}$$

where $\{W_i^+\}_{i=1}^k$ and $\{W_j^-\}_{j=k+1}^n$ are analytic bases of the stable/unstable manifolds of $x = \pm\infty$, respectively.

Theorem (Alexander, Gardner, Jones)

The Evans Function enjoys the following properties:

- $D(\lambda)$ is analytic to the right of the essential spectrum.
- Degree of roots of $D(\lambda)$ match multiplicity of $\sigma_P(L)$.

Remark

Since $D(\lambda)$ is analytic in right half plane we can use winding number computations to test for spectral stability.

Linearized equations

$(\hat{v}, \hat{u}_1, \hat{u}_2, \hat{B}_2)$

$$\begin{aligned}
v_t + v_x - u_{1x} &= 0 \\
u_{1t} + u_{1x} + (-a\gamma \hat{v}^{-\gamma-1} v + (1/\mu_0)(\hat{B}_2 B_2))_x &= \tau(u_{1x}/\hat{v} - \hat{u}_{1x} v/\hat{v}^2)_x \\
u_{2t} + u_{2x} - (I/\mu_0) B_{2x} &= \mu(u_{2x}/\hat{v} - \hat{u}_{2x} v/\hat{v}^2)_x \\
\tilde{\alpha}_t + \tilde{\alpha}_x - I u_{2x} &= (\sigma\mu_0)^{-1}(B_{2x}/\hat{v} - \hat{B}_{2x} v/\hat{v}^2)_x,
\end{aligned}$$

where $\tilde{\alpha} = \hat{v} B_2 + v \hat{B}_2$, so that $B_2 = (\tilde{\alpha} - \hat{B}_2 v)/\hat{v}$.

Eigenvalue problem

$$\lambda v + v' - u_1' = 0$$
$$\lambda u_1 + u_1' - (h(\hat{v})v/\hat{v}^{\gamma+1})' = -(\hat{B}_2(\tilde{\alpha} - \hat{B}_2 v)/(\mu_0 \hat{v}))' + \tau(u_1'/\hat{v})'$$
$$\lambda u_2 + u_2' - (I/\mu_0)(\tilde{\alpha}/\hat{v} - \hat{B}_2 v/\hat{v})' = \mu(u_2'/\hat{v} - \hat{u}_2' v/\hat{v}^2)'$$
$$\lambda \tilde{\alpha} + \tilde{\alpha}' - I u_2' = (\sigma\mu_0)^{-1}(\hat{v}^{-1}(\tilde{\alpha}/\hat{v} - \hat{B}_2 v/\hat{v})' - \hat{B}_2' v/\hat{v}^2)',$$

$$\begin{aligned} h(\hat{v}) &= -\hat{v}^{\gamma+1}(\tau \hat{u}_1'/\hat{v}^2 - a\gamma\hat{v}^{-\gamma-1}) \\ &= -\hat{v}^{\gamma+1}(\tau \hat{v}'/\hat{v}^2 - a\gamma\hat{v}^{-\gamma-1}) \\ &= -\hat{v}^{\gamma+1}(\hat{v}^{-2}(\hat{v}(\hat{v}-1) + a\hat{v}^{1-\gamma} - a\hat{v} + (2\mu_0\hat{v})^{-1}((B_{2-} + I\hat{u}_2)^2 - \hat{v}^2 B_{2-}^2)) - a\gamma\hat{v}^{-\gamma-1}). \end{aligned}$$

Integrated coordinates

$$u(x) = \int_{-\infty}^{x} u_1(z)dz, \quad w = \int_{-\infty}^{x} u_2(z)dz,$$

$$V = \int_{-\infty}^{x} v(z)dz, \quad \text{and } \alpha = \int_{-\infty}^{x} \tilde{\alpha}(z)dz$$

$$\begin{aligned}
\lambda V' + V'' - u'' &= 0 \\
\lambda u' + u'' - (h(\hat{v})V'/\hat{v}^{\gamma+1})' &= -(\hat{B}_2(\alpha' - \hat{B}_2 V')/(\mu_0 \hat{v}))' + \tau(u''/\hat{v})' \\
\lambda w' + w'' - (I/\mu_0)(\alpha'/\hat{v} - \hat{B}_2 V'/\hat{v})' &= \mu(w''/\hat{v} - \hat{u}_2' V'/\hat{v}^2)' \\
\lambda \alpha' + \alpha'' - Iw'' &= (\sigma\mu_0)^{-1}(\hat{v}^{-1}((\alpha' - \hat{B}_2 V')/\hat{v})' - \hat{B}_2' V'/\hat{v}^2)'.
\end{aligned}$$

Integrated coordinates

Integrating from $-\infty$ to x we obtain

$$
\begin{aligned}
\lambda V + V' - u' &= 0 \\
\lambda u + u' - h(\hat{v})V'/\hat{v}^{\gamma+1} &= -\hat{B}_2(\alpha' - \hat{B}_2 V')/(\mu_0 \hat{v}) + \tau(u''/\hat{v}) \\
\lambda w + w' - (I/\mu_0)(\alpha'/\hat{v} - \hat{B}_2 V'/\hat{v}) &= \mu(w''/\hat{v} - \hat{u}_2' V'/\hat{v}^2) \\
\lambda \alpha + \alpha' - I w' &= (\sigma\mu_0)^{-1}(\hat{v}^{-1}(\alpha'/\hat{v} - \hat{B}_2 V'/\hat{v})' - \hat{B}_2' V'/\hat{v}^2).
\end{aligned}
$$

Eigenvalue problem, $\sigma = \infty$

$u_3 \equiv B_3 \equiv 0$, $\sigma = \infty$,

$$
\begin{aligned}
\lambda V + V' - u' &= 0 \\
\lambda u + u' - h(\hat{v})V'/\hat{v}^{\gamma+1} &= -\hat{B}_2(\alpha' - \hat{B}_2 V')/(\mu_0 \hat{v}) + \tau(u''/\hat{v}) \\
\lambda w + w' - (I/\mu_0)(\alpha'/\hat{v} - \hat{B}_2 V'/\hat{v}) &= \mu(w''/\hat{v} - \hat{u}_2' V'/\hat{v}^2) \\
\lambda \alpha + \alpha' - I w' &= 0.
\end{aligned}
$$

Eigenvalue problem, $\sigma = \infty$

$$(u, v, v', w, \mu w', \alpha)^T,\ u'' = \lambda V' + V'',\ K = I^2/\mu_0$$

$$u' = \lambda V + V'$$

$$V'' = \frac{\lambda \hat{v} V}{\tau} + \left(-\frac{h(\hat{v})}{\tau \hat{v}^\gamma} - \lambda + \frac{\hat{v}}{\tau} - \frac{\hat{B}_2^2}{\mu_0 \tau} \right) V' + \frac{\lambda \hat{v} u}{\tau} + \frac{I \hat{B}_2 w'}{\mu_0 \tau} - \frac{\lambda \hat{B}_2 \alpha}{\mu_0 \tau}$$

$$w'' = \left(\frac{\hat{u}_2'}{\hat{v}} + \frac{I \hat{B}_2}{\mu_0 \mu} \right) V' + \frac{\lambda \hat{v} w}{\mu} + \frac{\hat{v} w'}{\mu} - \frac{K w'}{\mu} + \frac{\lambda I \alpha}{\mu_0 \mu}$$

$$\alpha' = I w' - \lambda \alpha.$$

First order system, $\sigma = \infty$

$$W' = A(x,\lambda)W$$

$$A(x,\lambda) = \begin{pmatrix} 0 & \lambda & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{\lambda \hat{v}}{\tau} & \frac{\lambda \hat{v}}{\tau} & f(\hat{v}) - \lambda - \frac{\hat{B}_2^2}{\mu_0 \tau} & 0 & \frac{I\hat{B}_2}{\mu_0 \mu \tau} & -\frac{\lambda \hat{B}_2}{\mu_0 \tau} \\ 0 & 0 & 0 & 0 & \frac{1}{\mu} & 0 \\ 0 & 0 & \frac{\mu \hat{u}_2'}{\hat{v}} + \frac{I\hat{B}_2}{\mu_0} & \lambda \hat{v} & \frac{\hat{v}-K}{\mu} & \frac{\lambda I}{\mu_0} \\ 0 & 0 & 0 & 0 & \frac{I}{\mu} & -\lambda \end{pmatrix},$$

$$W = (u, V, V', w, \mu w', \alpha)^T, \; f(\hat{v}) = \tau^{-1}(\hat{v} - \hat{v}^{-\gamma} h(\hat{v})).$$

Numerical stability investigation: profile approximation

- STABLAB (quite general)
- bvp4c, bvp5c, bvp6c (Lobatto quadrature scheme)
- finite computational domain $[-L, L]$
- $L_\pm$ chosen experimentally, $|U(\pm L_\pm) - U_\pm| <$ error
- projective boundary conditions $M_\pm(U - U_\pm) = 0$

Approximation of the Evans function

Polar coordinate method (Humpherys-Zumbrun)

$$\mathcal{W} = r\,\Omega,$$

$$\mathcal{W} = W_1 \wedge \cdots \wedge W_k$$

$$\Omega = \omega_1 \wedge \cdots \wedge \omega_k$$

$$D(\lambda) = \mathcal{W}^- \wedge \mathcal{W}^+|_{x=0} = \det(W_1^-, \dots, W_k^-, W_{k+1}^+, \dots, W_N)|_{x=0}.$$

$$\mathcal{W}^-(-L_-) \sim e^{-\mu L_-}(R_1^- \wedge \cdots \wedge R_k^-)$$

$$\mu = \mathrm{Trace} A_-|_{U(A_-)}$$

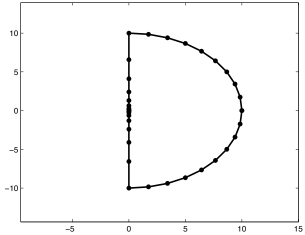
Shooting and initialization

- ode45 (adaptive 4th-order Runge-Kutta-Fehlberg)
- Error tolerance: AbsTol=1e-8, RelTol=1e-6
- Kato's ODE
- Second-order algorithm (Zumbrun)

Winding number computation

$$S := \partial\big(B(0,R) \cap \{\Re\lambda \geq 0\}\big)$$

- 20 points taken quadratic in modulus
- winding number calculation
- require relative error be less than 0.2
- Rouché's Theorem



HF convergence

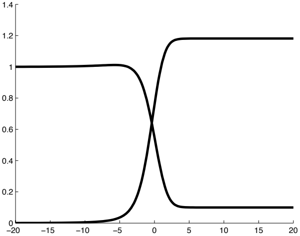
Proposition

$$\lim_{|\lambda|\to\infty} \frac{\tilde{D}(\lambda)}{e^{\alpha\lambda^{1/2}}} = C \text{ uniformly on } \Re\lambda \geq 0,$$

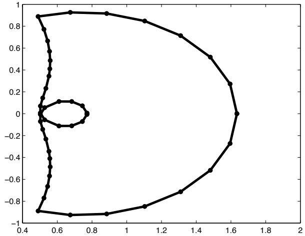
where α and C are constants.

$$\log \tilde{D}(\lambda) = \log C + \alpha\lambda^{1/2} \quad |\lambda| >> 1.$$

Transverse Evans function



(a)



(b)

Figure: Typical transverse Evans function output, parameter values $\gamma = 5/3$, $I = 0.6$, $B_+ = 1.4$, and $\mu_0 = 1$. In Figure (a) we display the nonmonotone profile. In Figure (b) we display the winding number computation.

Large Amplitude limit

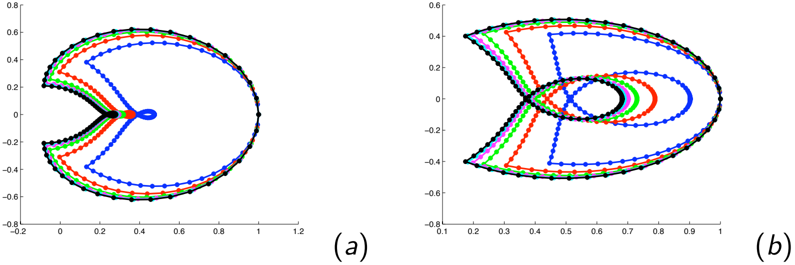


Figure: Large-amplitude limits, parameters $K = 2$, $J = 1$, $\gamma = 5/3$. In Figure (a), we display the image of the semicircle under $\tilde{D}$ for a Lax 2-shock in two-rest point configuration in the $a \to 0$ limit, $a = 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}$. where $a = 10^{-8}$ corresponds to Mach number $\approx 10,954$. Convergence of contours appears to occur at $a \sim 10^{-6}$. or Mach number $\approx 1,095$. In Figure (b), for the same sequence of a-values, we display the images under the transverse Evans function, again suggestive of convergence.

Large amplitude limits

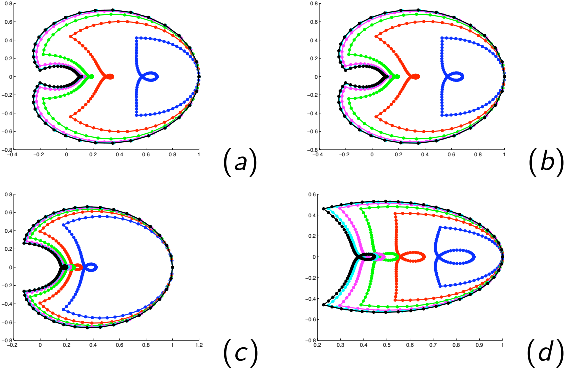


Figure: Large amplitude limits, parameters $K = 0.7$, $J = 0.5$, and $a = 10^{-1}, 10^{-2}, \ldots, 10^{-k}$, getting smaller as necessary to see what appear to be convergence to a limit. (a). Lax 2-shock, v_2 to v_1. (b). Intermediate Lax 2-shock, v_3 to v_1. (c). overcompressive 1-2 shock, v_4 to v_1. (d). Transverse Evans study for (c). In each case, we appear to obtain convergence at $a = 10^{-7}$, corresponding to Mach number $\approx 3,817$.

Experiments

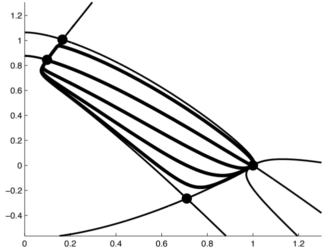
$$(\gamma, K, J, v_+, \mu_0) \in \{7/5, 5/3\}$$
$$\times \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 1.05, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0\}$$
$$\times \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0\}$$
$$\times \{0.1\}$$
$$\times \{1.0\}.$$

$$(\gamma, v_+, I, B_{2+}, \mu_0) \in \{7/5, 5/3\}$$
$$\times \{0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 10^{-1}, 10^{-2}\}$$
$$\times \{0.2, 0.4, 0.6, 0.8, 1.2, 1.4, 1.6, 1.8, 2.0\}$$
$$\times \{0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0\}$$
$$\times \{1.0\}.$$

Typical computation time around 1 minute

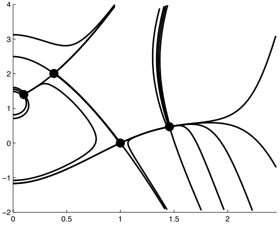
Experiments: overcompressive family

$$(\gamma, K, J, a, \mu_0) \in \{7/5, 5/3\}$$
$$\times \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95\}$$
$$\times \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0\}$$
$$\times \{a_1, a_2, a_3, a_4, a_5\}$$
$$\times \{1.0\}.$$



Experiments: undercompressive

$$(\gamma, v_+, K, J) \in \{7/5, 5/3\}$$
$$\times \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$$
$$\times \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$$
$$\times \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$$



The take away

- The system appears stable (known instabilities for multi-D model)
- Organizing the phase space important (knowing way around)
- adaptive mesh
- parallel again in future

Interesting directions

- Finite electrical resistivity
- Three dimensional profiles
- Full nonisentropic case
- Multidimensional stability (in x)