COURSE ANNOUNCEMENT, Spring 2015

Title: Stability of viscous shock waves and beyond

(Primary) Instructor: Kevin Zumbrun

Keywords: *Stability and bifurcation of viscous shock waves, numerical Evans function evaluation and spectra of differential operators, Evans vs. Lopatinski determinants and relations between viscous and inviscid stability, modulation of spatially periodic waves.*

Description: Stability of traveling waves and other special solutions of time-evolutionary systems is a rich and fascinating mathematical topic with applications to biology, chemistry, optics, population genetics, fluid dynamics, and elasticity, among other fields. In an appropriate reference frame, such solutions appear as equilibria, or rest points, of the system. Hence, one may begin by examining the spectra of the generator L of the formal linearization $u_t = Lu$ of the system about an equilibrium: an autonomous, but in general variable-coefficient differential operator. One may then examine the relation between spectral stability or instability linearized and nonlinear time-evolutionary stability in the sense of decay or growth of an initial perturbation as time goes to infinity.

For systems for which the linearized operator L has a *spectral gap*, i.e., strict separation between its spectra and the imaginary axis, the latter steps are relatively straightforward, analogous to Lyapunov theory for ordinary differential equations, with decay or growth at time-exponential rate. However, in cases such as viscous shock waves, Cahn–Hilliard phase-transitions, or spatially periodic wave trains on the line, for which L has essential spectra accumulating at the imaginary axis, hence no spectral gap, these steps become quite delicate, with decay at most at time-algebraic rate. Techniques developed to handle this scenario include the weighted norm method of Sattinger (scalar shocks and phase-transition fronts) and the renormalization methods of Bricmont-Kupiainen and Schneider, (phase-transition fronts and periodic wave trains, respectively), and collaborators. However, these are limited to the situation that the asymptotic behavior of perturbations involves at most one slow (i.e., convective-diffusive) mode, and in particular do not typically apply to viscous shock solutions of *systems*.

In this course, we present a third set of techniques developed by the instructor and collaborators for the treatment of stability of viscous shocks, which applies to all three cases mentioned above (see, eg., Howard and collaborators in the Cahn–Hilliard case and Barker-Johnson-Noble-Rodrigues-Zumbrun in the periodic case), but without limitations on the number of time-asymptotic slow modes. Based on inverse Fourier/Laplace transform estimates on linearized behavior combined with an "instantaneous projection" scheme detecting nonlinear cancellation, this approach yields nonlinear stability of viscous shock waves in the system case arising in physical applications such as compressible gas dynamics, viscoelasticity, and magnetohydrodynamics, under a natural (necessary and sufficient) set of assumptions on the spectra of the linearized operator L about the wave. With suitable modifications, it has given also extensions in the Cahn–Hilliard and spatially periodic settings: notably the first proof of nonlinear stability of periodic waves of the Kuramoto–Sivashinsky equation and related dissipative systems with conservation laws.

We discuss also the complementary problem of checking, analytically or numerically, the associated spectral stability conditions, with an emphasis on the *Evans function* and its efficient numerical approximation. In addition, we give a variety of physical applications to stability and bifurcation. The bulk of the lectures will be given by primary instructor Kevin Zumbrun, with mini-courses of 2-4 hours lecture or more presented by a number of guest speakers, extending the core topics in various directions.

Invited speakers and topics:

Texier: *Bifurcation of shock and detonations*
Barker: *Numerical Evans computation*
Metivier: *Hyperbolic initial-boundary-value problems*
Rodrigues: *Modulation of periodic wave trains*
Sandstede: *Periodic defect solutions and their stability*

Prerequisites: Familiarity with basic (real and complex) analysis, differential equations, and Fourier/Laplace transform.